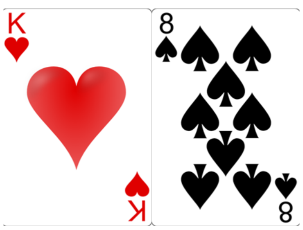
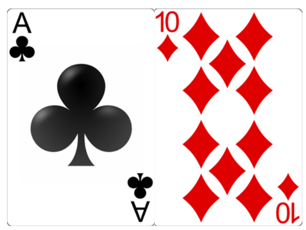
Introduction & Rules

Blackjack is the most popular casino card game, based upon the goal of having a hand totaling more points than the dealer's hand. The game starts out by having each *player*, including the *dealer*, being dealt two cards (one of which is turned face down and referred to as the **hole card**). After that, the game revolves around the sum of the two cards (and any that are added after the initial two), with face cards counting 10 points, aces either counting 1 **or** 11 points, and numerical cards counting towards their corresponding amount of points.

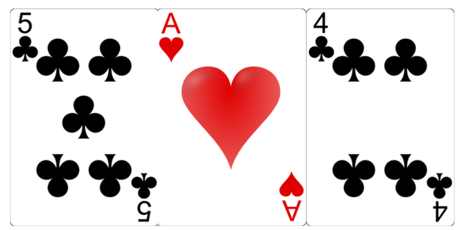
For example, the point totals for the following sets of cards are as follows: (*suit irrelevant*)



18 points



Either **11 or 21** points (a special case called *Blackjack*)



Either **10 or 20** points

In the case above, being dealt 21 points in the first two cards, called a *blackjack* or a *natural*, will guarantee an automatic win unless the dealer also achieved a blackjack.

After the initial dealing, each player, and then the dealer, is given a *turn* in which he/she may perform one of the following actions:

- **Stand** - Player keeps the cards he has and doesn't request or draw another
- **Hit** - Player draws or requests another card (and more if he wishes)
- **Split** - If the player has a pair, or any two 10-point cards, then he may double his bet and separate his cards into two individual hands. The dealer will automatically give each card a second card. Then, the player may hit, stand, or double normally. However, when splitting aces, each ace gets only one card.
- **Double-down** - Player doubles his bet and gets one, and only one, more card.
- **Surrender** - The player forfeits half his wager, keeping the other half, and does not play out his hand. This option is only available on the initial two cards.

Once all turns have been taken, the dealer flips over the hole card. If the dealer's total is less than 17, they must hit until it is greater than or equal to 17. After all actions are taken, there are two possible *win conditions*:

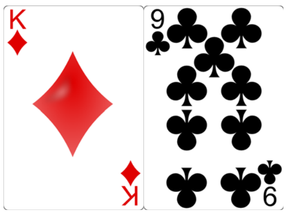
- If the dealer goes over 21 points, then any player who didn't already bust will win.
- If the dealer does not bust, then the higher point total between the player and dealer will win.

Because players are forced to go first, the situation can arise where the player loses their bets to the dealer even if the dealer would have lost anyway. This creates the root of blackjack's popularity with casinos: its house advantage. Depending on the amount of decks used, as well as the inclusion of specific rules, called *house rules*, it can seem very difficult to gain an edge as a player. However, despite these the odds, players can usually achieve a net gain if they follow a basic strategy, diminishing the chances of busting or losing to the dealer.

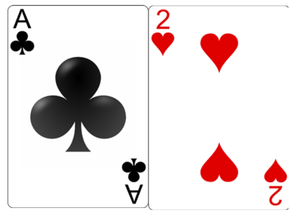
Sample Game

The proceedings for a sample game of 3 players (including a dealer) would be as follows:

The dealer would draw a pair of cards to every player (including him/herself):



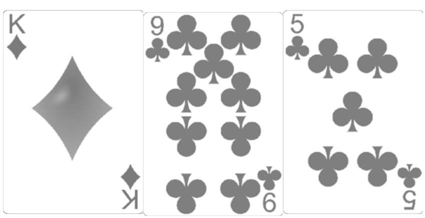
Player 1: **19** points



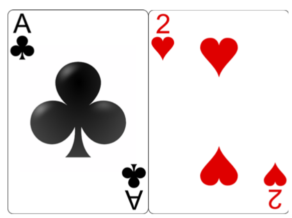
Player 2: **3 or 13** points

Dealer: **10** points exposed

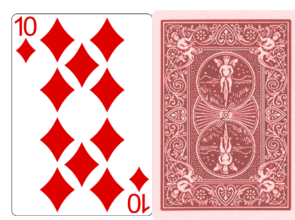
Because no players achieved *blackjack* on the first deal, the game proceeds normally. Player 1 takes their turn, deciding arbitrarily to *hit*. Unfortunately, they got dealt a *5 of Clubs*, driving their hand total to **24** points and causing them to go out, or *bust.*



Player 1: **24** points (*bust*)

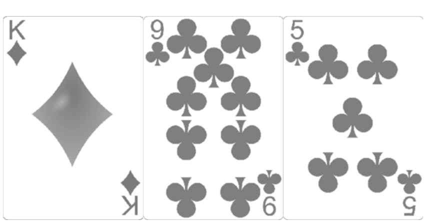


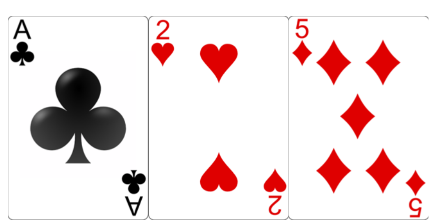
Player 2: **3 or 13** points

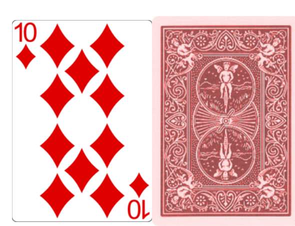


Dealer: **10** points exposed

After *Player 1*'s turn, *Player 2* takes their turn, deciding to be *hit* due to their low card-total and the flexibility of the hand with an ***ace***. They got dealt a *5 of diamonds,* bringing their hand total up to **8 or 18** points.





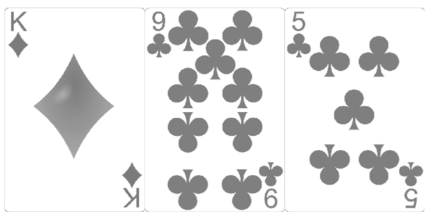


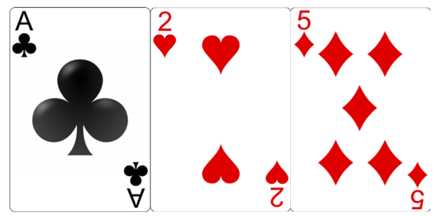
Player 1: **24** points (*bust*)

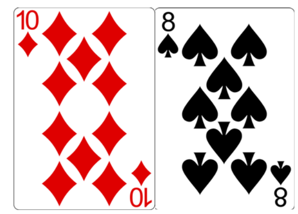
Player 2: **8 or 18** points

Dealer: **10** points exposed

After *Player 2* got dealt the *5 of diamonds*, they decide to *stand*, making the turn go to the dealer. They flip over the upside-down card, revealing an *8 of spades.*







Player 1: **24** points (*bust*)

Player 2: **18** points

Dealer: **18** points

Because the dealer's hand totals up to a quantity greater than 17, they *stand*. In the end, *Player 2* and the dealer have the same total (**18 points**). This results in a *push* for *Player 2*, who gets all of their bet back, and a *loss* for *Player 1*, who loses their bet to the house.

Computational Model

In order to simulate the proceedings of random blackjack games, our team developed a computational model based upon the principle of **Monte Carlo methods**, which rely on repeated random sampling to obtain numerical results. As a game, blackjack is an especially good candidate for Monte Carlo simulation due to its simplistic rules and gamestate in spite of the complex and diverse strategies available. In our case, we used the programming language called Java to simulate rounds of blackjack 10,000,000 times using two main strategies: threshold decision-making, and lookup-table based decision making.

The threshold decision-making strategy uses the simple check of whether the current maximum total of a hand is *less than* a given threshold value. If it is, then the hand is *hit*, and gains a new card. This process repeats until the card total is above the threshold.

On the other hand, the lookup-table strategy employs a mathematically-derived table of the best possible actions in given scenarios, as depicted below:

Player hand	**Dealer's face-up card**									
	2	3	4	5	6	7	8	9	10	A
Hard totals (excluding pairs)										
17–20	S	S	S	S	S	S	S	S	S	S
16	S	S	S	S	S	H	H	H	H	H
15	S	S	S	S	S	H	H	H	H	H
13–14	S	S	S	S	S	H	H	H	H	H
12	H	H	S	S	S	H	H	H	H	H
11	D	D	D	D	D	D	D	D	D	H
10	D	D	D	D	D	D	D	D	H	H
9	H	D	D	D	D	H	H	H	H	H
5–8	H	H	H	H	H	H	H	H	H	H
Soft totals										
	2	3	4	5	6	7	8	9	10	A
A,8–A,9	S	S	S	S	S	S	S	S	S	S
A,7	S	D	D	D	D	S	S	H	H	H
A,6	H	D	D	D	D	H	H	H	H	H
A,4–A,5	H	H	D	D	D	H	H	H	H	H

A,2–A,3	H	H	H	D	D	H	H	H	H	H
Pairs										
	2	3	4	5	6	7	8	9	10	A
A,A	SP	SP	SP	SP	SP	SP	SP	SP	SP	SP
10,10	S	S	S	S	S	S	S	S	S	S
9,9	SP	SP	SP	SP	SP	S	SP	SP	S	S
8,8	SP	SP	SP	SP	SP	SP	SP	SP	SP	SP
7,7	SP	SP	SP	SP	SP	SP	H	H	H	H
6,6	SP	SP	SP	SP	SP	H	H	H	H	H
5,5	D	D	D	D	D	D	D	D	H	H
4,4	H	H	H	SP	SP	H	H	H	H	H
2,2–3,3	SP	SP	SP	SP	SP	SP	H	H	H	H

H: Hit, *SP: Split,* *D: Double-down,* *S: Stand*

The specific strategy outlined above was defined from the *WizardOfOdds* website, on their page about 4-8 Deck Blackjack Strategy. Because it provides the action with the highest chances of winning for each scenario, it is the optimal strategy when looking at each hand on a case-by-case basis and without considering hand composition.

Parameters

- Surrendering is disallowed
- The dealer stands on a soft 17
- A new deck is used for each round, and is shuffled before use
- The simulation is performed 10,000,000 times
- In the first simulation (***Figure 2***), cards are drawn from 2 full decks

The final model's code is available as a GitHub Repository at:

- *https://github.com/jazevedo620/Blackjack-Simulation*

<u>Or by following the QR Code Below:</u>

Results (after 10,000,000 simulations)

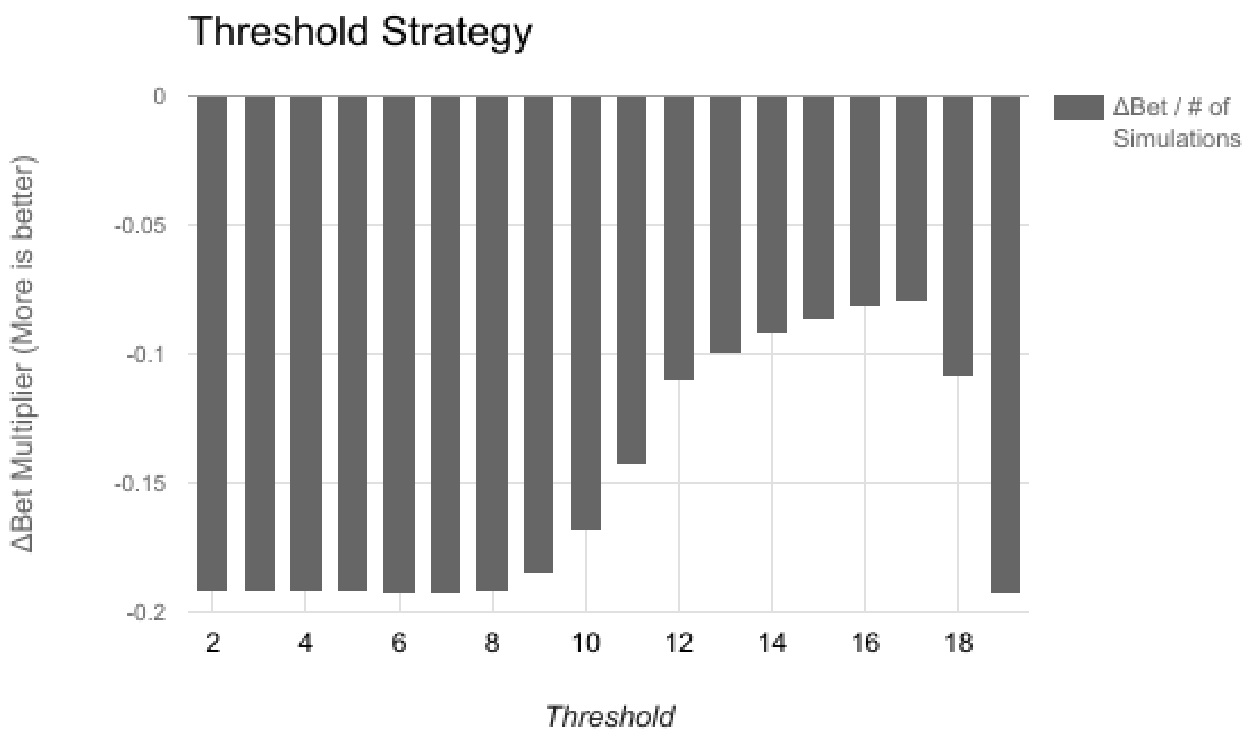


Figure 2

As is evident in *Figure 2*, using the threshold strategy results in a net loss of betting money over time, even using the best threshold, x = 17, where each game results in the loss of 8.006% of the original bet. The curve seen in the graph reflects the nature of the threshold strategy. Having a low threshold results in many losses simply due to a higher dealer hand total. However, having too high of a threshold quickly results in many *busts*. Taking the best-case threshold strategy and graphing it alongside the optimal strategy paints a clear picture of which one is superior:

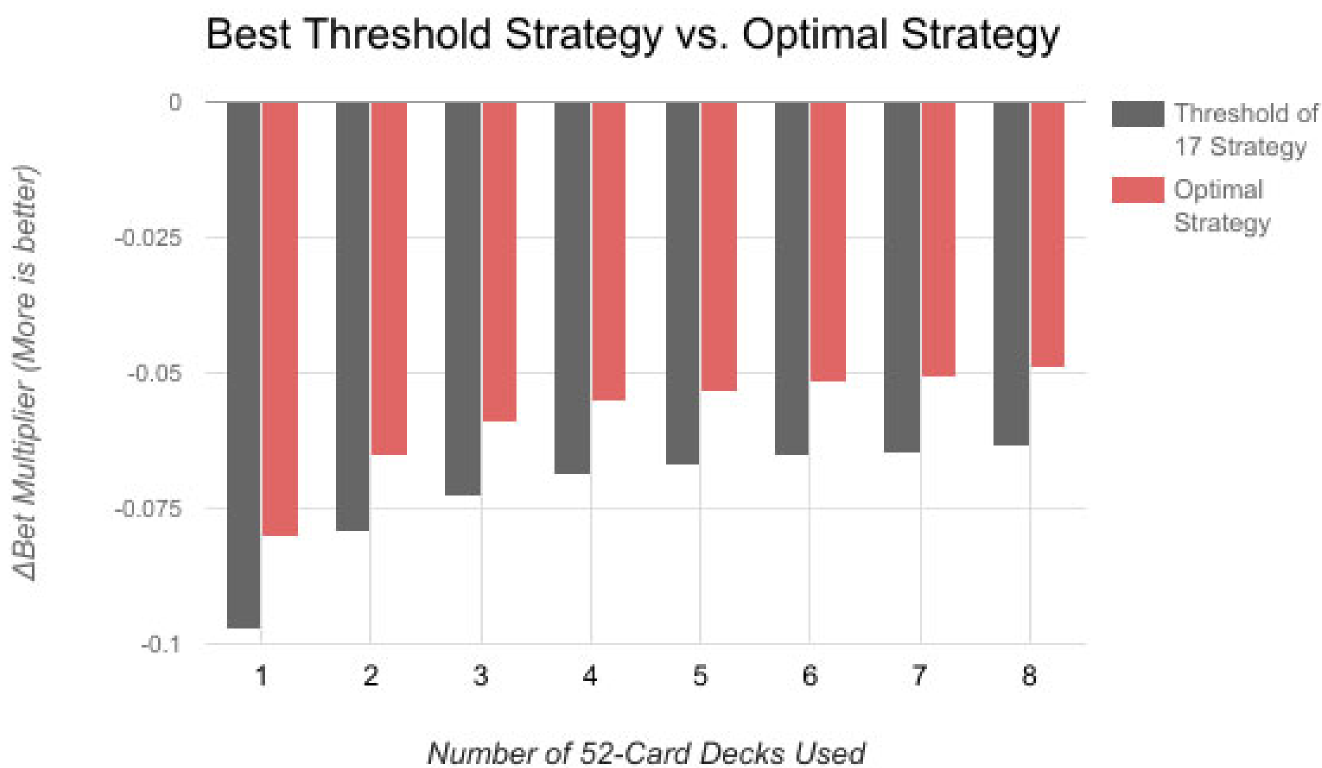


Figure 3

In *Figure 3*, the Optimal Strategy was the clear winner over even the best that the Threshold Strategy had to offer. Interestingly enough, however, the more decks used, the more of an advantage (or rather, the less of a disadvantage) the player had, which goes against the assumption that less decks increases the odds of blackjack (due to the proportional change of 1 card to the total card amount). This is likely because the optimal strategy was *optimized* and designed for 4-8 decks, as it says in its specifications. If the lookup table was redefined for fewer decks, the trend may have been different.

Mathematical Model

Going back to the look-up table defined in the *Computational Model* section, the entire table was generated using compounding probability. That is, every decision has a multitude of other possible decisions that could come afterwards. To simplify matters, we narrowed down the situation such that:

- Cards are drawn from an *infinite* pool of random cards
- The specific situation is where:
    - The values of cards in the player's hand total to **14 points**
    - The exposed card in the dealer's hand is worth **9 points**

Card dealt		Player result	Bust chance if hit		Dealer result	Action	Chance of *Bust*
Ace		**15**	**53.85%**		**10 or 20**	Stand	N/A
2		**16**	**61.54%**		**11**	Hit	**0%**
3		**17**	**69.23%**		**12**	Hit	**30.76%**
4		**18**	**76.92%**		**13**	Hit	**38.46%**
5		**19**	**84.62%**		**14**	Hit	**46.15%**
6		**20**	**92.31%**		**15**	Hit	**53.85%**
7		**21**	**100%**		**16**	Hit	**61.54%**
8		*Bust*	N/A		**17**	Stand	N/A
9		*Bust*	N/A		**18**	Stand	N/A
10		*Bust*	N/A		**19**	Stand	N/A
J		*Bust*	N/A		**19**	Stand	N/A
Q		*Bust*	N/A		**19**	Stand	N/A
K		*Bust*	N/A		**19**	Stand	N/A

		46.154% of *Bust*				(stand on 17)	**38.46%** chance of *Bust*

Probabilities were generated using the premise that each of the 13 face values have an equal chance of being drawn:

$$\{A,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ J,\ Q,\ K\}$$

For example, when looking at the third card dealt to the player in the second content column, it can be seen that not all of the face values will result in a playable hand. The card values

$$\{8,\ 9,\ 10,\ J,\ Q,\ K\}$$

will all result in a *bust*. Because of this, they can be expressed as a probability ratio, such that, on the first player hit, the situation results in a

$$\frac{6}{13} = 46.154\%$$

chance of a *Bust.*

Conclusion

Because of its status as a game of chance, blackjack can have infinitely many factors influencing the outcome of a game. Invisible and uncontrollable ones such as the value of the next card in the deck, as well as the dealer's hole card, all play a role. However, as you've seen by now, certain strategies can be used to lessen the amount or risk involved, mitigate losses, and end up with a net gain. The basic strategy we've explored here is able to be mathematically proven and helps to prevent busting or losing to the dealer. Still, we haven't even scratched the surface. With advanced techniques and advantage play, such as *card counting* and *compositional look-up tables*, one can boost their odds of winning. Even though the luck factor is ultimately unavoidable, hopefully, with this information, you can get out there and beat the odds!

Sources Used

I. Official Blackjack Rules retrieved from BicycleCards.com: http://www.bicyclecards.com/how-to-play/blackjack/

II. Optimal Strategy Lookup table retrieved from WizardOfOdds.com: https://wizardofodds.com/games/blackjack/strategy/4-decks/